EXAMPLE 2 ➤ Find x if $0.258 = 10^x$. Verify your solution numerically.

SOLUTION By definition, x, the exponent of 10, is the logarithm of 0.258.

$x = \log 0.258 = -0.5883\ldots$ By calculator. Do not round.

CHECK $10^{-0.5883\ldots} = 0.258$

which checks. ◄

Examples 3, 4, and 5 show you how to verify numerically the three properties of logarithms.

EXAMPLE 3 ➤ Show numerically that $\log(7 \cdot 9) = \log 7 + \log 9$. Explain how this property agrees with the definition of logarithm.

SOLUTION

$\log(7 \cdot 9) = \log 63 = 1.7993\ldots$

$\log 7 + \log 9 = 0.8451\ldots + 0.9542\ldots = 1.7993\ldots$ Calculate without rounding.

$\therefore \log(7 \cdot 9) = \log 7 + \log 9$

This equality agrees with the definition because

$(7 \cdot 9) = (10^{0.8451}\ldots)(10^{0.9542})$

$= 10^{0.8451\ldots+0.9542}$ Add the exponents. Keep the same base.

$= 10^{1.7993}$

$\therefore \log(7 \cdot 9) = 1.7993\ldots$ The logarithm is the exponent of 10. ◄

EXAMPLE 4 ➤ Show numerically that $\log \frac{51}{17} = \log 51 - \log 17$. Explain how this property agrees with the definition of logarithm.

SOLUTION

$\log \dfrac{51}{17} = \log 3 = 0.4771\ldots$

$\log 51 - \log 17 = 1.7075\ldots - 1.2304\ldots = 0.4771\ldots$ Calculate without rounding.

$\therefore \log \dfrac{51}{17} = \log 51 - \log 17$

This equality agrees with the definition because

$\dfrac{51}{17} = \dfrac{10^{1.7075}}{10^{1.2304}} = 1^{1.7075\ldots -1.2304}$ Subtract the exponents. Keep the same base.

$= 10^{0.4771\ldots}$

$\therefore \log \dfrac{51}{17} = 0.4771\ldots$ The logarithm is the exponent of 10. ◄